

but the main changes are in his second volume. Some of these are quite minor: spelling, middle names, notation, small numerical errors, and they would not justify this review. But, besides many other changes of greater significance, two sections of the book have been extensively revised to bring in recent research.

The first of these topics is Euclid's algorithm and now includes some new research by Philipp, Dixon and Heilbronn.

The largest change is an account of the new Strassen-Schönhage "fast Fourier multiplication" algorithm including

**THEOREM S.** *It is possible to multiply two  $n$ -bit numbers in  $O(n \log n \log \log n)$  steps.*

It is clear that the Mersenne prime and related industries will have to completely retool to take advantage of this new technology. With such retooling, and the faster machines now available, we can expect a new prime  $2^p - 1$  to show itself one of these days.

To make room for this new material while keeping the same pagination, certain topics in the first printing, such as gaps between successive primes, are deleted—through no fault of their own.

Naturally, Knuth is driving his publisher bananas with all these changes, and he begins his compilation here, amusingly, with "The author wishes to encourage everyone to stop doing any further research, so that he may finish writing the other volumes." Why such a mild way of trying to halt the tide of research; why not simply command it, as did his ancestor, the King?

D. S.

41[4].—JAMES W. DANIEL & RAMON E. MOORE, *Computation and Theory in Ordinary Differential Equations*, W. M. Freeman & Co., San Francisco, 1970, viii + 172 pp., 24 cm. Price \$7.50.

Because of their training, physicists and engineers tend to stick to analytic methods as long as possible, before submitting their problems to a computer. In their treatment of problems in differential equations, they may sometimes end up by offering them to the computer in a form less suitable than the original version. On the other hand, the present generation of numerical analysts suffers from the opposite extreme: as computers grow faster and memories become larger, numerical analysts spend less and less effort on preparing a problem in differential equations analytically. They simply grind it through the mill. The book under review attempts to counteract this tendency and to resubstitute mathematical analysis, at least in part, for computing by brute force.

For this purpose, the authors present a variety of analytic techniques which may be used to transform a given system of differential equations into one which is better suited for numerical solution. Their main criterion for that "suitability" is the straightness of the "flow" defined by the system through its vector field. Thus, the transformations they study are designed to straighten out large local variations or curvatures in this "flow". This common strategy unifies the treatment in this book which is subdivided into sections on "a priori global transformations" and "a posteri-

ori transformations". The last term denotes efforts to facilitate the computation of a neighboring solution of a given (or computed) solution. The techniques range from old classical approaches to the use of local or moving coordinate systems which combine analytic and computational methods. The authors state no theorems nor do they give rigorous derivations; instead, they demonstrate the rationale of the various lines of attack and try to illuminate them by well-chosen examples. Naturally, they refer to the relevant literature for a more thorough treatment.

The third part of the book is preceded by a section on basic facts about differential equations (selected so as to prepare for the later discussion) and a section on numerical methods which, very briefly, gives some of the principal concepts. In this section, a variety of classes of numerical methods is discussed (including some not so well-known ones). The authors make an interesting attempt at an evaluation of their relative merit. Again, the reader has to resort to the quoted literature for any hard-core facts, if he does not already know them. Throughout the book, the main emphasis rests on initial-value problems. In the first two parts of the book, some attention is given to boundary-value problems.

While these parts may serve as a welcome guide to the literature, it is the third part on transformations which makes the book a very valuable contribution to numerical analysis as well as to scientists who have to solve differential equations. Though many of the techniques described will need some further development and computational experience (as the authors freely admit), it is hoped that their exhibition will stimulate worthwhile efforts in that direction.

H. J. S.

42[7].—JACQUES DUTKA, *The Square Root of 2 to 1,000,000 Decimals*, ms. of 200 computer sheets + 1 supplementary page, deposited in the UMT file.

The decimal value of the square root of 2 to one million places is herein presented on 200 computer sheets, each containing 5000 decimal digits. A supplementary page gives the succeeding 82 decimal places.

This carefully checked calculation required a total of about 47.5 hours of computer time on the IBM 360/91 system at Columbia University.

Further details and pertinent background information have been given by the author in a paper [1] appearing elsewhere in this journal.

J. W. W.

1. JACQUES DUTKA, "The square root of 2 to 1,000,000 decimals," *Math. Comp.*, v. 25, 1971, pp. 927–930.

43[8].—H. LEON HARTER & D. B. OWEN, Editors, *Selected Tables in Mathematical Statistics*, Markham Publishing Co., Chicago, 1970, vii + 405 pp., 29 cm. Price \$5.80 cloth.

This book is Volume I of a projected series of volumes of mathematical tables prepared under the aegis of the Institute of Mathematical Statistics. A list of the tables and their authors follows: